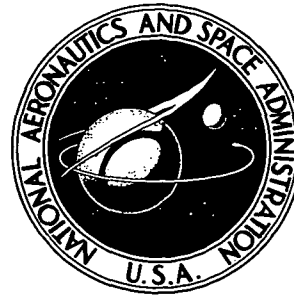


N72-33682

NASA TECHNICAL NOTE



NASA TN D-6942

NASA TN D-6942

CASE FILE
COPY

ISOSINGLET APPROXIMATION FOR NONELASTIC REACTIONS

by John W. Wilson

Langley Research Center

Hampton, Va. 23365

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1972

1. Report No. NASA TN D-6942		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle ISOSINGLET APPROXIMATION FOR NONELASTIC REACTIONS				5. Report Date October 1972	
				6. Performing Organization Code	
7. Author(s) John W. Wilson				8. Performing Organization Report No. L-8425	
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23365				10. Work Unit No. 503-10-04-01	
				11. Contract or Grant No.	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546				13. Type of Report and Period Covered Technical Note	
				14. Sponsoring Agency Code	
15. Supplementary Notes					
16. Abstract <p>It is not always feasible to provide complete sets of nonelastic reaction data for space-radiation shielding or radiation damage studies. The present work derives group theoretic relations between different combinations of projectile and secondary particles which appear to have a broad range of application. These relations are used to reduce the experimental effort required to obtain nuclear-reaction data for transport calculations. Implications for theoretical modeling are also noted, especially for heavy-heavy reactions.</p>					
17. Key Words (Suggested by Author(s)) High-energy transport Nuclear reactions Nonelastic reactions			18. Distribution Statement Unclassified - Unlimited		
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 16	
				22. Price* \$3.00	

ISOSINGLET APPROXIMATION FOR NONELASTIC REACTIONS

By John W. Wilson
Langley Research Center

SUMMARY

It is often necessary in high-energy transport calculations to approximate in some manner the cross sections for which data are not available. Most common are interpolation in energy or atomic number and mass. Relations of yet another kind can be established by using isospin symmetries of the strong force and a few simple assumptions about nuclear reactions and nuclear states.

The Serber transition operator is formed of two factors $\tau = \tau_e \tau_c$, the cascade and evaporation part, respectively. The cascade event is via strong interactions which conserve isospin, and the evaporation is isospin symmetry breaking.

Since τ_c proceeds via the strong force, then isospin rotation relates cascade cross sections of isospin conjugate states. That isospin conjugate states of the evaporation cross sections are not similarly related follows since τ_e is isospin symmetry breaking. Since the strong force is most cohesive for states of lowest isospin, however, it is reasonable to assume that the intermediate nuclear state R is an isosinglet (that is, an isospin scalar particle). Thus, the state R is unchanged under isospin rotation and the evaporation spectra are consequently unaffected by isospin rotations.

INTRODUCTION

High-energy hadron transport calculations require an extensive knowledge of the nonelastic cross sections for processes occurring within the transport media. Although considerable progress has been made toward attaining complete sets of experimental and theoretical data, they are as yet very incomplete. In order that transport calculations could proceed, approximate relations have often been developed to extend existing data into regions where cross sections are unknown (refs. 1, 2, 3, and 4).

It is well known that energetic nuclear production cross sections are smooth functions of A and Z for sufficiently large A . Standard procedures have developed which use this property in preparation of cross sections for transport calculations (ref. 1). This procedure has eliminated the need for performing experiments for many materials for the purpose of nucleon transport studies.

Methods have been developed for interpolation and extrapolation in primary energy (refs. 1, 3, and 4). They are based on the observation that the high-energy secondary production spectra, if plotted against the ratio of secondary energy to primary energy, are found to be nearly independent of primary energy (ref. 1). Although this property is expected to be true only between inelastic thresholds (refs. 2 and 3), detailed studies have shown it to be not unreasonable in extrapolating to very high energies over many such thresholds (ref. 4). Caution must be applied with this method since it applies only to the so-called "cascade" event and is not valid for the low-energy nucleons usually attributed to "evaporation" processes (refs. 1 and 5).

The present work will describe in detail extensions of yet another type. It generally occurs that experimental data are available for limited types of primary and secondary particles; for example, high-energy beams of neutrons are rarely available, and neutrons are not usually measured reliably in the final state. Thus, neutron data almost always must be inferred from measurements on other particles. Described herein is a derivation of relations based on an $SU(2)$ approximation of the nonelastic process. These relations have proven to be helpful in developing cross-section data for transport calculations (refs. 2, 3, and 5). The development uses the Serber two-step model (ref. 6), but the results may have validity in a broader sense.

SYMBOLS

A nucleon number

E entering-state label

F final-state label

F' transformed final-state label

F_c final cascade-particle-state label

F_e final evaporation-state label

F_1 intermediate scattering-state label

$I_y(\theta)$ isospin operator for rotation through angle θ about i_y

i, i_z, i', i'_z isospin quantum numbers, dimensionless

\hat{i}_y	unit vector along y-axis in isospin space
N	neutron number
P	proton number
p, n	nucleon-state labels, p for proton state and n for neutron state
R	residual excited-nuclear-state label
T	transition-operator matrix elements, amu^{-1}
u	general isospin rotation operator, dimensionless
K, Ω	energy and angle of secondary particles
δ_{ij}	Kronecker δ -function
η	phase factor
π^\pm, π^0	pion-state labels (superscript denotes charge)
τ	state transition operator, amu^{-1}
$ a\rangle$	vector in Hilbert space with label a that represents a complete set of quantum numbers
$\sigma_{X \rightarrow Y}$	double differential nonelastic cross section from entering state X to final state Y
$\sigma_{x,y}$	double differential nonelastic cross section for production of y -particle with x -projectile

Subscripts and superscripts:

c	cascade
E	entering

e	evaporation
F	final
F ₁	intermediate state
p	projectile
R	residual
s	strong
y	y-component
z	z-component
γ	electromagnetic
†	dagger over an operator denotes Hermitian conjugate

ELEMENTARY CONCEPTS

Serber Model

It will be assumed for the present purposes that the Serber two-step model (ref. 6) for high-energy nuclear reactions is substantially correct. The dynamical basis of this model has been discussed extensively elsewhere (refs. 6, 7, and 8 and references therein) and will not be repeated here. The cascade process of the first step proceeds via the strong interaction (the coulomb scattering peak is neglected) and is subject to the symmetries (in particular, isospin) of the strong force. The first-step transition operator τ_c is defined so that the transition matrix element is

$$T_c(F_1, E) = \langle F_1 | \tau_c | E \rangle \quad (1)$$

in state vector notation (ref. 9) where $|E\rangle$ and $|F_1\rangle$ denote the asymptotic entering and final states of the cascade process, respectively. In the model, it is assumed that the final cascade state contains one or more highly excited residual nuclei denoted by

$$|F_1\rangle = |R\rangle |F_c\rangle$$

where $|R\rangle$ is the residual nuclear state and $|F_c\rangle$ is the cascade particle state (that is, particles ejected by direct interaction). The excited nuclei decay through particle emission (evaporation) and finally γ -radiation. This deexcitation occurs through strong and electromagnetic interaction which breaks isospin invariance. The step-two deexcitation transition operator τ_e is related to the total T-matrix element of the reaction by

$$T = \langle F | \tau_e \tau_c | E \rangle = \sum \langle F | \tau_e | F_1 \rangle \langle F_1 | \tau_c | E \rangle \quad (2)$$

Since the Serber model conveniently divides into these two steps, one which is isospin invariant and the other which is isospin symmetry breaking, this reaction is easily analyzed.

Isospin, SU(2)

An outline of those properties of the representations of the isospin group in relation to observations of hadronic systems which are required is now presented. Additional detail on SU(2) and the strong interaction are to be found in reference 10. Let $u \in SU(2)$; then, for every transition operator τ_s of only strongly interaction particles

$$[\tau_s, u] = 0 \quad (3)$$

which is the statement of isospin invariance. The quantum numbers of the isospin group, i and i_z , are conserved by τ_s as follows:

$$\langle i', i'_z | \tau_s | i, i_z \rangle = T_s^i \delta_{ii'} \delta_{i_z i'_z} \quad (4)$$

where superscript i on T_s means it depends only on i and not on i_z . Electromagnetic interactions τ_γ depend on the projection i_z , and it follows that generally

$$[\tau_\gamma, u] \neq 0 \quad (5)$$

and isospin is not conserved.

Systems of hadrons are identified by isospin quantum numbers which belong to various isospin multiplets. The multiplicity determines the i state (that is, Multiplicity = $2i + 1$ for $i = 0, 1/2, 1, 3/2, \dots$) with the i_z component measured by the symmetry-breaking electromagnetic interaction (that is, electric charge). The symmetry operation of importance here is isospin rotations about the \hat{i}_y axis by π as denoted by $I_y(\pi)$. The following hadrons are of interest:

(1) Nucleon isodoublet

There are two charge states:

$$\text{Proton} \quad |p\rangle = |1/2, 1/2\rangle \quad (6a)$$

$$\text{Neutron} \quad |n\rangle = |1/2, -1/2\rangle \quad (6b)$$

which are related by

$$I_y(\pi)|p\rangle = |n\rangle \quad (6c)$$

$$I_y(\pi)|n\rangle = -|p\rangle \quad (6d)$$

(2) Pion isotriplet

There are three pion-charge states:

$$|\pi^+\rangle = |1, 1\rangle \quad (7a)$$

$$|\pi^0\rangle = |1, 0\rangle \quad (7b)$$

$$|\pi^-\rangle = |1, -1\rangle \quad (7c)$$

which satisfy

$$I_y(\pi)|\pi^\pm\rangle = -|\pi^\mp\rangle \quad (7d)$$

$$I_y(\pi)|\pi^0\rangle = -|\pi^0\rangle \quad (7e)$$

(3) Nuclei

Nuclei which are presently considered to be bound systems of nucleons are characterized by nucleon number A , proton number P , and neutron number N where

$$A = P + N \quad (8a)$$

$$i_z = (P - N)/2 \quad (8b)$$

It is observed that small isospin numbers are favored by the strong force; for example, ground-state nuclei have the smallest i consistent with i_z which gives

$$i = |i_z| = |P - N|/2 \quad (8c)$$

The operation $I_y(\pi)$ yields

$$I_y(\pi)|i, i_z\rangle = \eta|i, -i_z\rangle \quad (8d)$$

where η is a trivial phase factor. For nuclei with $P = A/2$

$$i = |P - N|/2 = |A - A|/4 = 0 \quad (8e)$$

which are isosinglet hadrons.

ISOSINGLET APPROXIMATION

Step One, Cascade

The cascade process consists of multiples of binary collisions within the target nucleus. The number of binary collisions with nucleons of the same type clearly depends on the number of nucleons of that type (that is, P or N , whichever is appropriate). The fractional number of collisions with protons is expected to be $\approx P/A$ and similarly for neutrons $\approx N/A$. Furthermore, if P and N are large, then $(P \pm 1)/A \approx P/A$ and $(N \pm 1)/A \approx N/A$ so that secondary production spectra are not expected to be rapidly varying functions of P and N . On this basis, the amplitudes for

$$p + C^{12} \rightarrow \text{Anything}$$

$$p + C^{13} \rightarrow \text{Anything}$$

differ by no more than

$$(7/13 - 6/12) \approx 8 \text{ percent}$$

for cascade processes. In the following it is assumed that the neighboring nuclei composed of combinations of $P \pm 1$ and $N \pm 1$ are indistinguishable in nonelastic reactions. Furthermore, only nuclei for which

$$P \approx N \approx A/2 \quad (9)$$

are considered and refer to the isosinglet approximation as the assumption that

$$P = N = A/2 \quad (10)$$

The entering state consists of two hadrons, one of which has $A \gg 1$ for which the isosinglet approximation is assumed to be valid; so the isospin quantum numbers of the entering state are those of the second hadron (hereafter referred to as the projectile and denoted by subscripts p). Hence, using the isosinglet approximation, for the entering state

$$\begin{aligned} |i_E, i_{E_Z}, E\rangle &= | |P-N|/2, (P-N)/2 \rangle |i_p, i_{p_Z}\rangle \\ &\approx |0, 0\rangle |i_p, i_{p_Z}\rangle \equiv |i_p, i_{p_Z}, E\rangle \end{aligned} \quad (11)$$

where isospin numbers of the states are now written explicitly.

The final state consists of the many hadrons which were expelled from the nucleus by the direct binary reactions of the cascade process and one or more heavy residual nuclei which are left in highly excited states. These excited nuclei subsequently decay through particle and γ emission. These residual nuclei, it is assumed have

$$P_R \approx N_R \approx A_R/2 \quad (12a)$$

$$A_R \gg 1 \quad (12b)$$

and, since the strong force favors states of small isospin, for residual nuclei

$$i_R = |P_R - N_R|/2 \approx 0 \quad (12c)$$

that is, an isosinglet approximation. The cascade particles (those expelled by binary collisions) have total isospin number i_c and projection i_{c_Z} . Hence, the final cascade-state configuration is

$$\begin{aligned} |i_{F_1}, i_{F_{1_Z}}, F_1\rangle &= |i_R, i_{R_Z}, R\rangle |i_c, i_{c_Z}, F_c\rangle \\ &\approx |0, 0, R\rangle |i_c, i_{c_Z}, F_c\rangle \equiv |i_c, i_{c_Z}, F_1\rangle \end{aligned} \quad (13)$$

The assumed isospin invariance of the cascade transition operator implies for $u = I_y(\pi)$

$$\begin{aligned} T_c(F_1, E) &= \langle F_1, i_c, i_{c_Z} | \tau_c | i_p, i_{p_Z}, E \rangle \\ &= \langle F_1, i_c, i_{c_Z} | u^\dagger u \tau_c u^\dagger u | i_p, i_{p_Z}, E \rangle \\ &= \langle F_1, i_c, -i_{c_Z} | \tau_c | i_p, -i_{p_Z}, E \rangle = T_c^{i_p} \delta_{i_p i_c} \delta_{i_{p_Z} i_{c_Z}} \end{aligned} \quad (14)$$

since $\tau_c = u\tau_c u^\dagger$, and in the last line of equation (14) u is taken as $I_y(\pi)$. Hence, the cascade transition matrix element depends only on total isospin of the projectile, and the final-cascade-particles isospin is equal to the projectile isospin.

Step Two, Evaporation

As noted earlier, the evaporation process is greatly dependent on electromagnetic forces which are not isospin invariant. The evaporation characteristic time is long ($\gg 10^{-23}$ sec) compared with the short time duration of the cascade event ($\leq 10^{-23}$ sec); hence, the excited residual nuclear states are comparatively stable and the residual nucleus at the end of the cascade $|R\rangle$ is far removed in time from the cascade interaction region. Thus, nontrivial matrix elements of the evaporation operator are taken between the excited residual nuclear state $|R\rangle$ and the final evaporation products $|F_e\rangle$ as

$$\langle F_e | \tau_e | R \rangle$$

The final state is naturally divided into two sets of particles: the cascade particles produced in direct binary reactions and the evaporation products of the residual excited nuclei which are denoted as follows

$$|F_e\rangle |F_c\rangle = |F\rangle$$

The full transition matrix element is

$$\begin{aligned} T(F, E) &= \langle F, i_F, i_{F_Z} | \tau_e \tau_c | i_p, i_{p_Z}, E \rangle \\ &= \sum_{F_1, i, i_Z} \langle F, i_F, i_{F_Z} | \tau_e | i, i_Z, F_1 \rangle \langle F_1, i, i_Z | \tau_c | i_p, i_{p_Z}, E \rangle \\ &= \sum_{F_1} \langle F, i_F, i_{F_Z} | \tau_e | i_p, i_{p_Z}, F_1 \rangle T_c^{ip}(F_1, E) \\ &= \sum_R \langle F_e | \tau_e | R \rangle T_c^{ip}(F_1, E) \end{aligned} \tag{15}$$

where $|F_e\rangle$ is the state of final evaporation products and $|R\rangle$ is the state of residual nuclei appearing in the final cascade state and is assumed to be an isosinglet. The assumption that $|R\rangle$ is an isosinglet effectively decouples the element $\langle F_e | \tau_e | R \rangle$ in isospin rotations.

FINAL RESULTS

It has been suggested that the nonelastic reaction is independent of the isospin of the target nucleus when

$$|P - N|/A \ll 1 \quad (16)$$

so the entering-state isospin quantum numbers can be taken as those of the projectile. Similarly it is argued that the residual nuclei after the cascade processes can be approximated by isosinglet states. The final state is thus divided into two sets of particles: the cascade particles produced in direct binary reactions and the evaporation products of the residual excited nuclei which are denoted as follows

$$|F_e\rangle|F_c\rangle = |F\rangle \quad (17)$$

Assuming the residual nuclei to be isosinglet effectively decouples the evaporation products from isospin rotations u as seen by combining equations (14) and (15) as follows

$$T(F', uE) = \sum_R \langle F_e | \tau_e | R \rangle T_C^{ip}(uF_1, uE) \quad (18)$$

where $|F'\rangle = |F_e\rangle|uF_c\rangle$ which follows from the assumption that $|R\rangle$ is an isosinglet, so that

$$u|R\rangle = |R\rangle \quad (19)$$

Again the evaporation process due to assumption (19) has been effectively decoupled in isospin rotations.

The cross section of these nonelastic processes is proportional to the T-matrix element squared. Equation (18) implies that the matrix element for the process $E \rightarrow F$ is numerically equal to the matrix element for $(uE) \rightarrow F'$. The assumed invariance of the cascade operator and the isosinglet approximation imply that

$$\sigma_{E \rightarrow F} = \sigma_{(uE) \rightarrow F'}$$

where $|F'\rangle = |F_e\rangle|uF_c\rangle$. These results are applied to specific scattering systems in the next section.

APPLICATION

Isosinglet-Isosinglet Reactions

When both hadrons of the entering state are isosinglets

$$u|E\rangle \equiv |uE\rangle = |E\rangle \quad (20)$$

$$i_c \approx i_p = 0 \quad (21)$$

Thus, from equation (18) (F' as in eq. (18))

$$T(F', uE) = \sum_R \langle F_e | \tau_e | R \rangle T_c^O(uF_1, E) \quad (22)$$

from which it may be deduced that the cascade particles which are related by

$$I_Y(\pi) |i, i_Z\rangle = \eta |i, -i_Z\rangle \quad (23)$$

have the same energy spectra. For reactions induced by deuterons or α -particles the following correspondence between measured parameters of secondary cascade particles may be noted:

$$\sigma_{\alpha, n}^c(K, \Omega) = \sigma_{\alpha, p}^c(K, \Omega)$$

$$\sigma_{\alpha, \pi^+}^c(K, \Omega) = \sigma_{\alpha, \pi^-}^c(K, \Omega)$$

$$\sigma_{\alpha, {}^3H}^c(K, \Omega) = \sigma_{\alpha, {}^3He}^c(K, \Omega)$$

These relations are especially important for high-energy secondary neutrons since they are rarely measured reliably.

Isosinglet-Isodoublet Reactions

Among the isodoublet hadrons which are used as projectiles are (p,n) and (${}^3He, {}^3H$) corresponding to $i_Z = (1/2, -1/2)$. The two possible entering states are denoted by $|E^+\rangle$ and $|E^-\rangle$ in an obvious notation with

$$I_Y(\pi) |E^-\rangle = \eta |E^+\rangle$$

where η is an unimportant phase factor assumed to be 1. Applying equation (18) (for $u \equiv I_Y(\pi)$)

$$T(F', E^-) = \sum_R \langle F_e | \tau_e | R \rangle T_c^{1/2}(uF_1, uE^+)$$

with F' as before. Hence the relations among secondary cascade particles for the two entering isospin states follow:

$$\sigma_{p,p}^c(K, \Omega) = \sigma_{n,n}^c(K, \Omega)$$

$$\sigma_{p,n}^c(K, \Omega) = \sigma_{n,p}^c(K, \Omega)$$

$$\sigma_{p,\pi^\pm}^c(K, \Omega) = \sigma_{n,\pi^\mp}^c(K, \Omega)$$

$$\sigma_{p,3H}^c(K, \Omega) = \sigma_{n,3He}^c(K, \Omega)$$

$$\sigma_{p,d}^c(K, \Omega) = \sigma_{n,d}^c(K, \Omega)$$

$$\sigma_{p,\alpha}^c(K, \Omega) = \sigma_{n,\alpha}^c(K, \Omega)$$

which are production cross sections for protons, neutrons, deuterons, tritons, helion, α -particles, and pions due to incident protons and neutrons. Since the intermediate residual excited nuclear states are assumed to be isosinglet nuclei, the following relations among secondary evaporation products are also obtained:

$$\sigma_{p,p}^e(K, \Omega) = \sigma_{n,p}^e(K, \Omega)$$

$$\sigma_{p,n}^e(K, \Omega) = \sigma_{n,n}^e(K, \Omega)$$

$$\sigma_{p,d}^e(K, \Omega) = \sigma_{p,d}^e(K, \Omega)$$

.

.

.

Similar results can be obtained for various entering-state configurations.

Comparison of Results

There are as yet insufficient experimental results to make comparisons with the present work. The only comparisons to be made are relations among results of model calculations of nonelastic reactions. Currently, the most complete model of nonelastic reactions is that of Bertini (ref. 11). The essential feature of the model of interest here is that it is a Serber model with isospin invariant reactions in the first step followed by a standard evaporation model (Dresner, ref. 12) for the second step. Extensive calculations have been made for reactions induced by p , n , π^\pm , and oversimplified α -particles. The cascade secondary particles allowed in the Bertini calculations (ref. 11) are p , n , π^\pm , and π^0 . According to the present work, the high-energy proton and neutron spectra for α -induced reactions should be equal in the isosinglet approximation. Results obtained from the α model of reference 13 (Gabriel and others) for reactions in ^{27}Al using the Bertini calculations (ref. 11) are shown in figure 1. These results agree to within the statistical accuracy of the Monte Carlo method.

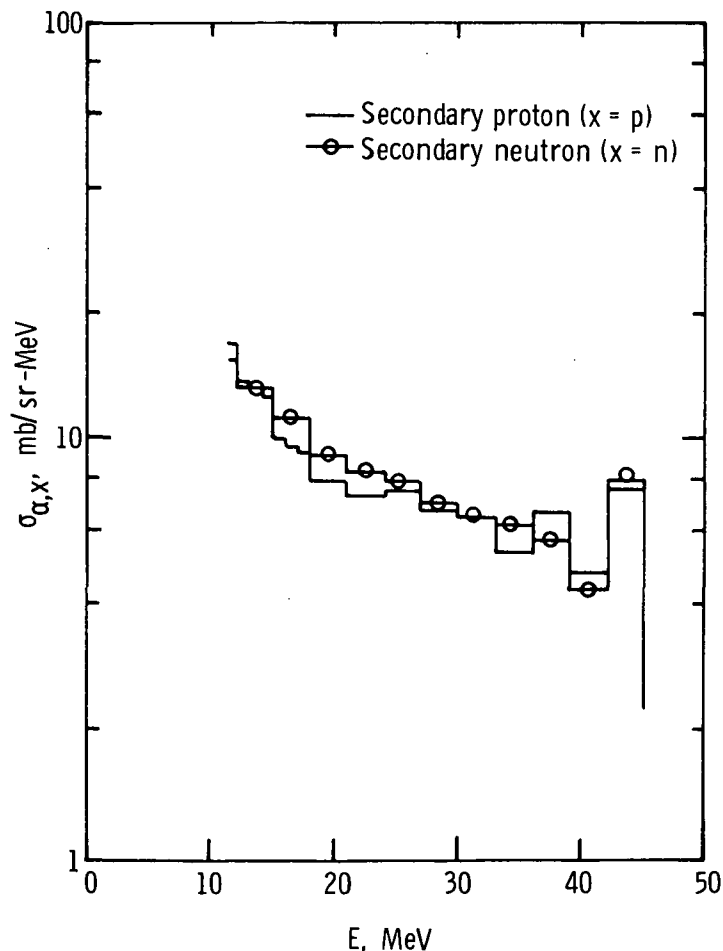


Figure 1.- Comparison of secondary-production spectra for the α - ^{27}Al model reaction of reference 13. These spectra should be equal under the isosinglet approximation.

TABLE I. - QUANTITIES WHICH ARE EXACTLY RELATED BY SU(2) IN
NONELASTIC REACTION. (DATA TAKEN FROM REF. 11.)

Cascade multiplicity	
p- ¹⁶ O reaction	n- ¹⁶ O reaction
Proton 2.051	Neutron 2.045
Neutron 1.471	Proton 1.490
π^+ 0.268	π^- 0.280
π^- 0.081	π^+ 0.080
π^0 0.180	π^0 0.165

TABLE II. - QUANTITIES RELATED THROUGH THE ISOSINGLET APPROXIMATION
APPLIED TO ENTERING STATE. (DATA TAKEN FROM REF. 11.)

Cascade multiplicity	
p- ²⁷ Al reaction	n- ²⁷ Al reaction
Proton 2.291	Neutron 2.426
Neutron 1.861	Proton 1.695
π^+ 0.263	π^- 0.273
π^- 0.087	π^+ 0.080
π^0 0.165	π^0 0.160

TABLE III. - QUANTITIES RELATED THROUGH THE ISOSINGLET APPROXIMATION
APPLIED TO THE INTERMEDIATE RESIDUAL EXCITED NUCLEAR STATE.
(DATA TAKEN FROM REF. 11.)

Evaporation multiplicity	
p- ¹⁶ O reaction	n- ¹⁶ O reaction
Proton 0.841	Proton 0.794
Neutron 1.114	Neutron 0.866
Deuteron 0.091	Deuteron 0.115
Triton 0.025	Triton 0.025
Helion (³ He) 0.029	Helion (³ He) 0.036
Alpha 0.341	Alpha 0.389

Bertini calculations for protons and neutron-induced reactions in ^{16}O and ^{27}Al for 1 GeV energy are given in reference 11. A direct application of equation (18) for the ^{16}O reaction yields the results given in table I. Note that these results are exact in the sense that the Bertini calculations are an implementation of the Serber model. The results in table II assume that ^{27}Al is well approximated by an isosinglet state. Results for the isosinglet approximation as applied to the intermediate excited nuclear state are shown in table III. Based on these model calculations, the isosinglet approximation appears to be adequate for the description of nonelastic processes.

CONCLUDING REMARKS

It is interesting to see that simple algebraic relations can be derived for systems which react with the complexity of nonelastic reactions. In particular, when simple systems interact with heavy target nuclei, the final reaction products do not depend strongly on the target isospin structure but do depend strongly on the projectile. This dependence appears to be in part due to the tendency of the strong force to be most attractive for systems with small isospin numbers. The exceptional cohesiveness of states of small isospin is a property which can be of great consequence in the study of heavy-heavy nonelastic reactions.

It is to be emphasized that these relations are not dependent on the detailed dynamics except to the extent that the strong force is $\text{SU}(2)$ invariant and the isosinglet approximation is valid. In the case of the α - ^{27}Al reaction model of Gabriel, Santoro, and Alsmiller, in which the model α is totally unrealistic, results that are $\text{SU}(2)$ consistent occur. It should be noted that it is only the dynamical description of the α model (four unbound nucleons) which is unrealistic.

From a nuclear-physics point of view, the present results yield a dynamic independent probe for the nonelastic process. Disagreement among secondary-particle parameters for energies above that to be associated with nuclear temperature would indicate either $\text{SU}(2)$ breaking terms in the first-step Serber model or more preferably violation of the isosinglet approximation for the residual excited nuclear state.

The immediate consequence of the present results is to help in obtaining data for transport calculations. These results will reduce the number of necessary experiments by a factor of two. The derived relations are especially important for neutrons which are difficult to measure reliably and are rarely available as intense high-energy beams for nuclear-physics experiments.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., September 12, 1972.

REFERENCES

1. Leimdorfer, M.; and Barish, J.: A Method of Representing Two-Dimensional Distributions for Use in Monte Carlo Calculations. ORNL-4002 (Contract W-7405-Eng-26), U.S. At. Energy Comm., Jan. 1967. (Available as NASA CR-82502.)
2. Foelsche, T.; Mendell, Rosalind; Adams, Richard R.; and Wilson, John W.: Measured and Calculated Radiation Levels Produced by Galactic and Solar Cosmic Rays in SST Altitudes and Precaution Measures To Minimize Implications at Commercial SST-Operations. NASA paper prepared for French-Anglo United States Supersonic Transport Meeting (Paris, France), Mar. 3, 1969.
3. Wilson, John W.; Lambiotte, Jules J., Jr.; Foelsche, Trutz; and Filippas, Tassos A.: Dose Response Functions in the Atmosphere Due to Incident High-Energy Protons With Application to Solar Proton Events. NASA TN D-6010, 1970.
4. Gabriel, T. A.; Alsmiller, R. G., Jr.; and Guthrie, M. P.: An Extrapolation Method for Predicting Nucleon and Pion Differential Production Cross Sections From High-Energy (> 3 GeV) Nucleon-Nucleus Collisions. ORNL-4542, U.S. At. Energy Comm., May 1970.
5. Lambiotte, Jules J., Jr.; Wilson, John W.; and Filippas, Tassos A.: Proper 3C: A Nucleon-Pion Transport Code. NASA TM X-2158, 1971.
6. Serber, R.: Nuclear Reactions at High Energies. Phys. Rev., vol. 72, no. 11, Dec. 1, 1947, pp. 1114-1115.
7. Goldberger, M. L.: The Interaction of High Energy Neutrons and Heavy Nuclei. Phys. Rev., vol. 74, no. 10, Nov. 15, 1948, pp. 1269-1277.
8. Harp, G. D.; Miller, J. M.; and Berne, Bruce J.: Attainment of Statistical Equilibrium in Excited Nuclei. Phys. Rev., vol. 165, no. 4, Jan. 20, 1968, pp. 1166-1169.
9. Merzbacher, Eugen: Quantum Mechanics. John Wiley & Sons, Inc., 1963.
10. Burhop, E. H. S., ed.: High Energy Physics. Vol. I. Academic Press, Inc., 1967.
11. Bertini, Hugo W.: Preliminary Data From Intranuclear-Cascade Calculations of 0.75-, 1-, and 2-GeV Protons on Oxygen, Aluminum, and Lead, and 1-GeV Neutrons on the Same Elements. ORNL-TM-1996, U.S. At. Energy Comm., Dec. 1967.
12. Dresner, L.: EVAP - A FORTRAN Program for Calculating the Evaporation of Various Particles From Excited Compound Nuclei. ORNL-TM-196, U.S. At. Energy Comm., Apr. 11, 1962.
13. Gabriel, T. A.; Santoro, R. T.; and Alsmiller, R. G., Jr.: An Approximate High-Energy Alpha-Particle Nucleus Collision Model. Nucl. Sci. & Eng., vol. 44, no. 1, Apr. 1971, pp. 104-110.



POSTMASTER: If Undeliverable (Section 158
Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

**SCIENTIFIC AND TECHNICAL INFORMATION OFFICE
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C. 20546**